ADAPTIVE AUTOREGRESSIVE MODEL WITH WINDOW EXTENSION VIA EXPLICIT GEOMETRY FOR IMAGE INTERPOLATION

Qingyun Wang, Jiaying Liu*, Wenhan Yang, Zongming Guo

Institute of Computer Science and Technology, Peking University, China, 100871

ABSTRACT

In this paper, we propose a novel adaptive autoregressive (AR) model constructed with an explicit geometry based extended window for image interpolation. Geometric features are chosen as criterions to include more useful pixels. These features are estimated explicitly and guide the interpolation window to extend adaptively. To characterize the piecewise stationary of images, the patch-geodesic distance based similarity is proposed and modulated into the adaptive AR model. For increasing the precision of the parameter estimation, a weighted ridge regression based estimation is employed. With the estimation, the multicollinearity between parameters, which occurs in piecewise stationarity conditions, is eliminated. Experimental results demonstrate that the proposed method is better than or competitive with state-of-the-art interpolation methods in both objective and subjective quality evaluations.

Index Terms— Image interpolation, autoregressive model, window extension, weighted ridge regression.

1. INTRODUCTION

Image interpolation is a technique that rescales a low-resolution (LR) image to a high-resolution (HR) version. In the past decades, there have been a great number of works on the image interpolation. In general, image interpolation techniques can be classified into three categories: conventional methods, explicit interpolation methods and implicit interpolation methods. Conventional methods, such as Bilinear and Bicubic interpolations, apply a convolution to every pixel of the LR image. However, as these methods do not capture the fast varying property around edges and texture structures, aliasing, blurring and ringing artifacts occur in high frequency regions.

To better model the essential property of edge regions, the explicit adaptive interpolation methods spring up. These methods utilize the structural information in an explicit way. They estimate the directions of edges and isophotes, and interpolate in these directions. Wang and Ward [1] proposed an isophote-based interpolation method. A parallelogram adapted to the local isophote is constructed and a directional bilinear interpolation is employed to generate HR pixels along the parallelogram lattice rather than the image lattice.

In [2], a segment adaptive gradient angle interpolation (SAGA) utilizes the information of isophotes and represents a three-step interpolation method, generating HR pixels in the local isophote rather than the image lattice. Yang *et al.* [3] proposed an improved fine-grained isophote model with consistency constraints. These explicit adaptive methods achieve superior performance to traditional ones. However, when it comes to the images containing lots of complex structures, the isophotes estimated with these methods tend to be unpredictable, which increases the variation of the interpolation.

Instead of the explicit utilization of local structures, the last kind of methods, the implicit adaptive methods, embed these structures into the objective function and interpolate by optimizing it. A typical achievement is the autoregressive (AR) model. Li and Orchard [4] proposed a new edge-directed interpolation (NEDI). They estimate the model parameters according to the geometric duality and reconstruct the HR pixels with corresponding LR scale parameters. In [5], the soft-decision adaptive interpolation (SAI) adds a cross-direction constraint in the AR model and estimates HR pixels jointly rather than separately, which contributes to the better performance upon NEDI. However, these algorithms are generally based on the stationarity assumption in images, which does not hold on account of the diversification of natural images. Aiming to model the non-stationarity of image signals, our previous work IPAR [6] and an adaptive general scale interpolation [7] proposed similarity probability models. Due to getting rid of the global stationarity assumption, weighted AR methods raise the precision of interpolation and better characterize the piecewise stationarity of images.

In this paper, we follow the AR-based framework and propose a novel interpolation method combining explicit and implicit methods. In our method, geometric features, such as isophotes and curvatures, are estimated explicitly and used as the criterions that guide the interpolation window to extend adaptively. The patch-geodesic distance based similarity is deployed to characterize the piecewise stationary of images. And the weighted ridge regression based block estimation raises the precisions of the AR parameters estimation and function minimization. Comprehensive experiments demonstrate that our method achieves desirable performance, no matter in objective or subjective quality evaluations

The rest of the paper is organized as follows: Section 2 reviews the AR model. Section 3 describes the proposed

adaptive interpolation algorithm built on geometry based widow extension AR model. Experimental results are presented in Section 4. Finally, concluding remarks are given in Section 5.
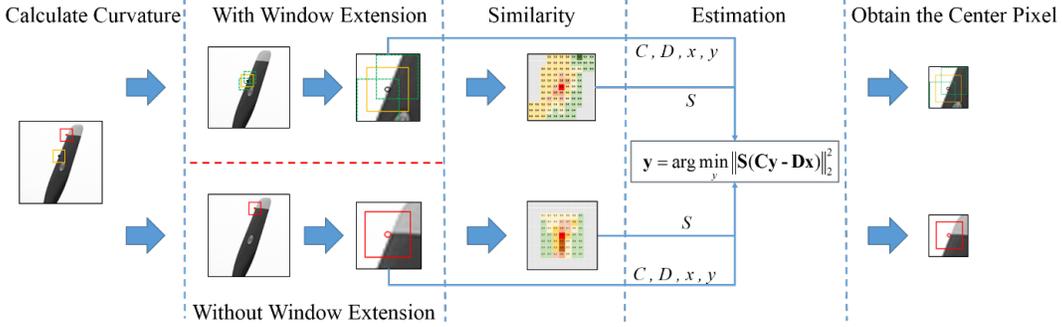
Fig.1. The flow chart of the proposed interpolation algorithm

2. AUTOREGRESSIVE (AR) MODEL

The autoregressive model is an effective tool for the image modeling. It models and predicts missing pixels based on the locality and stationarity. For instance, in the form of AR model, pixels in images are estimated by their adjacent neighbors with certain weights as follow:

$$I(m,n) = \sum_{(x,y)\in\Omega} \varphi(x,y)I(m+x,n+y)+\varepsilon, \quad (1)$$

where $\varphi(x,y)$ is the model parameter. Ω is the adjacent neighbor of the pixel $I(m,n)$. ε is the fitting error.

To estimate precisely, two kinds of model parameters, those in cross directions and diagonal directions, are set up and estimated respectively in a rectangle local window. More details about the parameter estimation will be elaborated in Sec. 3.4.

However, two issues limit the performance of the interpolation. First, AR models in a rectangle window are not adaptive to the local structures. Second, the stationarity assumption may be violated even in a very small region where the image signal fluctuates dramatically.

3. THE PROPOSED INTERPOLATION ALGORITHM

In this section, we present a new implicit statistical image interpolation method. First, a novel AR model based on geometry-aware adaptive window-extension is performed. And we introduce the patch-geodesic distance to define the similarity of two pixels. Then, we put forward our implicit image interpolation algorithm. Finally, the parameter estimation is described based on weighted ridge regression.

3.1. Geometry-Aware Adaptive Window-Extension

Geometry information in the local region provides hints for the interpolation. A useful clue is the self-similarity along the isophote: patches located along the same isophote are similar with each other, and the information contained in adjacent similar patches may benefit the interpolation of target pixel. So the model estimates the isophote and then extends the interpolation window in the isophote direction. However, the exceptions of high curvature should be paid attention to. As shown in Fig.1, the center pixel of the window is located at the confluence of two edges. Due to the high curvature, any extension introduces the pixels whose AR parameters are not consistent with the central one. Thus, before the extension, we calculate the curvature of the center pixel and eliminate the extension in the high curvature situation. The curvature is calculated as:

$$K(x,y) = \left| \left(-F_x^2 F_{yy} + 2F_x F_y F_{xy} - F_y^2 F_{xx}\right)/\left(F_x^2+F_y^2\right)^{3/2} \right|, (2)$$

where K is the curvature of the center pixel. F_x and F_y are the partial derivatives of pixel $I(x,y)$, while F_{xx}, F_{xy} and F_{yy} stand for two-order-partial derivatives of pixel $I(x,y)$. If $K < T$, we choose a direction and extend the window. In our algorithm, the threshold T is adaptive to the angle θ between isophote and horizontal line as follow:

$$T = \begin{cases} 1/\tan\theta, & \theta \geq 45° \\ 1/\cot\theta, & \theta < 45° \end{cases}. \quad (3)$$

As for the extension details, there are 8 directions to be chosen as the extension direction. For convenience, we define the set of direction angles M as $\{0°, 27°, 45°, 63°, 90°, 117°, 135°, 153°\}$. And we use the θ mentioned above to estimate extension direction and, like the method in [2], make use of the definition of isophote to calculate θ. If the intensity function is treated as locally planar, intensities at arbitrary locations can be estimated based on the collected data and their first-order derivatives:

$$I(x,y) = I(m,n) + F_x(x-m) + F_y(y-n). \quad (4)$$

Specially,

$$I(m+u,n+u\alpha) = I(m,n) + F_x u + F_y u\alpha. \quad (5)$$

Here the isophote is approximated locally with a line of constant intensity such that:

$$I(m+u,n+u\alpha) = I(m,n). \quad (6)$$

Given the original equalities, Eqs. (5) and (6) reduce to:

$$\alpha = -F_x / F_y. \quad (7)$$

And θ can be calculated as:

$$\theta = \begin{cases} 180° - \tan^{-1}\alpha, \alpha \geq 0 \\ -\tan^{-1}\alpha, \alpha < 0 \end{cases}. \quad (8)$$

Finally, we use θ to find the most approximate angle in M as the extension direction.

For the window after extension, we define two kinds of windows: the root window and the leaf window, representing the original window and the extension window respectively. As shown in Fig.2, extension starts from the root window. And leaf windows are added to be part of the window along a certain extension direction. In the end, an irregular window that contains pixels in both the root window and leaf windows is built.
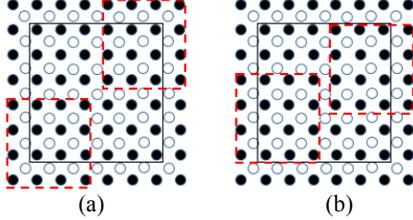
Fig. 2. Illustrations for the window extension results. Black solid lines represent the root windows, while red dotted lines represent the leaf windows. (a) The window extended in 45°. (b) The window extended in 27°.

3.2 .Patch-Geodesic Distance Based Similarity

Since the global stationarity is not valid in some regions, the piecewise stationarity in natural images is regarded as the basis of the AR parameter estimation. To better characterize the piecewise stationarity in the local region, a novel similarity metric is proposed, combining the spatial distance with the pixel intensity distance and modulating them into the AR model.

The similar idea in [8] guides the design of the new metric. Rather than accumulate the distance of successive pixels, we sum up the distance of successive patches. Incorporating the pattern similarity, the new metric effectively reflects whether two pixels are in the same region and share similar AR model parameters.

Let c denote the center pixel and x denote one pixel in the interpolation window. The patch-geodesic distance $D(x,c)$ is defined as the minimum value of the patch differences along all paths:

$$D(x,c) = \min_{P \in \rho_{x,c}} d(P), d(P) = \sum_{i=2}^{n} \sum_{j \in N} \left| I(p_{i,j}) - I(p_{i-1,j}) \right|, \quad (9)$$

where $\rho_{x,c}$ stands for the set of all paths connecting x and c. $N = \{1,2,\ldots,8\}$ contains all the neighboring indices of 8-neighbors. $p_{i,j}$ is the j-th neighbor of the i-th pixel in P and the $I(p_{i,j})$ stands for the intensity of $p_{i,j}$. After obtaining patch-geodesic distance, we convert it to the similarity metric:

$$p(x,c) = \exp(-D(x,c) / \beta), \quad (10)$$

where β is a user-defined parameter controlling the importance of distance weight.

The similarity probability between each pixel and the center pixel y_c is calculated throughout the local window. Let x_i and y_i be LR and HR pixels in local window. The similarity probability between x_i and y_c is written as p_i^L, and similarly, p_i^H represents the similarity probability between y_i and y_c.

3.3. Similarity Modulated Block Estimation

With the adaptive window extension and the similarity model mentioned above, we propose a novel AR interpolation method. In our algorithm, we use weighted block estimation to estimate the missing HR pixels. Two sets of parameters $\boldsymbol{a} = \{a_t\}$ and $\boldsymbol{b} = \{b_t\}$ ($t=1, 2, 3, 4$) describe the model parameter in two directions. The AR equations for a pixel in the diagonal direction and the cross direction can be represented as:

$$z_i = \sum_{t=1}^{4} a_t z_{i \otimes t} + \varepsilon_i^{\otimes}, z_i = \sum_{t=1}^{4} b_t z_{i \oplus t} + \varepsilon_i^{\oplus}, \quad (11)$$

where z_i refers to either LR pixels x_i or HR pixels y_i in the local window W. $z_{i \otimes t}$ and $z_{i \oplus t}$ are the diagonal and cross-direction neighbors around z_i pixel. $\varepsilon_i^{\otimes}$ and $\varepsilon_i^{\oplus}$ refer to random perturbations independent of spatial locations and image signal levels..

As mentioned in Sec. 3.2, the similarity probability p_i indicates the consistency of AR model parameters. Thus, the model fitting error at each pixel z_i is weighted by p_i. Taking this into consideration, we minimize the fitting error of the pixels in the window W by solving the linear least squares problem in Eq. (12)

$$\min_{\{y_i\}} \left\{ \sum_{i \in W} \left[p_i^H \left(y_i - \sum_{t=1}^{4} a_t y_{i \otimes t} \right) \right]^2 + \sum_{i \in W} \left[p_i^L \left(x_i - \sum_{t=1}^{4} a_t x_{i \otimes t} \right) \right]^2 \right.$$
$$\left. + \lambda \sum_{i \in W} \left[p_i^H \left(y_i - \sum_{t=1}^{4} b_t y_{i \oplus t} \right) \right]^2 \right\}, \quad (12)$$

where λ is the Lagrange multiplier. The method to estimate $\boldsymbol{a}$ and $\boldsymbol{b}$ will be elaborated in Sec. 3.4. Only the center pixel y_c is output in one block estimation process.

Let $\mathbf{x}$ and $\mathbf{y}$ be the vectors consisting of the LR and HR pixels in W respectively. Let $\mathbf{C}$ and $\mathbf{D}$ be the vectors consisting of the covariance (a_t or b_t) between pixels. Let $\mathbf{S}$ be the diagonal matrix composed of the similarity probability (p_i^L or p_i^H). We can deduce the objective function in Eq. (12) to a vector form:

$$\mathbf{y} = \arg \min_y \|\mathbf{S}(\mathbf{Cy} - \mathbf{Dx})\|_2^2 . \quad (13)$$

Then a close-formed resolution can be obtained:

$$\mathbf{y} = (\mathbf{C}^T \mathbf{S}^2 \mathbf{C})^{-1} \mathbf{C}^T \mathbf{S}^2 \mathbf{Dx} . \quad (14)$$

3.4. Weighted Ridge Regression with Parameter Estimation

As Eq. (11), the model parameters a_t and b_t have the same form, and can be estimated with the same method. For convenience, we take a_t to elaborate our estimation method.

Like Eq. (12), we can estimate a_t by solving the linear least squares problem:

$$\min_{a_t} \sum_{i \in W} \left[p_i^L \left(x_i - \sum_{t=1}^{4} a_t x_{i \otimes t} \right) \right]^2 . \quad (15)$$

And we can also deduce the objective function in Eq. (15) to a vector form:

$$\mathbf{a} = \arg \min_a \|\mathbf{W_l}(\mathbf{Aa} - \mathbf{x})\|_2^2, \quad (16)$$

where $\mathbf{W}_l$ is the diagonal matrix composed of the similarity probability (p_i^L). $\mathbf{A}$ is a 36×4 matrix whose k-th row is composed of diagonal neighbors of x_k ($x_{k \otimes t}$).

However, in the presence of typical piecewise stationarity, the patterns of the model parameters are simple. Multicollinearity may exist between the model parameters. It results in the expansion of variance, which means the undesirable precision of the estimation. A method in [9] introduces weighted ridge regression (WRR) to reduce the influence of multicollinearity. The weighted ridge regression modulates weights into the regression to value the reliability of each sample, leading to more reliable estimations. The weights used for WRR are the same as those in Sec.3.3. Combined with WRR, Eq. (16) can be modeled as:

$$\mathbf{a} = \arg\min_a \left\| \mathbf{W}_l (\mathbf{Aa} - \mathbf{x}) \right\|_2^2 + \lambda \left\| \mathbf{a} \right\|^2. \quad (17)$$

And Eq. (17) can be converted to a vector form:

$$\mathbf{a} = (\mathbf{A}^T \mathbf{W}_l^2 \mathbf{A} + \lambda \mathbf{I})^{-1} \mathbf{A}^T \mathbf{W}_l^2 \mathbf{x} . \quad (18)$$

Similarly,

$$\mathbf{b} = (\mathbf{B}^T \mathbf{W}_l^2 \mathbf{B} + \lambda \mathbf{I})^{-1} \mathbf{B}^T \mathbf{W}_l^2 \mathbf{x} , \quad (19)$$

where $\mathbf{B}$ is a 36x4 matrix whose k-th row is composed of cross-direction neighbors of x_k ($x_{k \oplus t}$).

4. EXPERIMENTAL RESULTS

The proposed interpolation algorithm is implemented on MATLAB 7.6 platform and compared with conventional Bicubic interpolation method and three state-of-the-art interpolation methods: NEDI [4], SAI [5] and IPAR [6]. We test the proposed algorithm on a large image set, including the Kodak database, many standard test images and some piecewise smooth images.

Table 1 PSNR(dB) results of five interpolation methods

Images	Bicubic	NEDI	SAI	IPAR	Proposed
Lena	34.01	33.72	34.76	34.79	**34.80**
Cameraman	25.51	25.44	25.99	26.06	**26.13**
Monarch	31.93	31.80	33.08	**33.34**	33.31
Airplane	29.40	28.00	29.62	30.05	**30.06**
House	22.20	21.74	22.28	22.33	**22.39**
Bike	25.41	25.25	26.28	**26.31**	**26.31**
Lighthouse	**26.97**	26.37	26.70	26.76	26.88
Barbara	**24.46**	22.36	23.55	23.10	24.34
Average	27.49	26.84	27.78	27.84	**28.03**

To compare the objective quality of different interpolation methods, the original HR images are first directly downsampled by a factor of two to generate the input LR images. Then, different interpolation methods are applied to enlarge the input LR images to those in their original resolutions. Table 1 tabulates the PSNR results of the five interpolation methods on several images in our experiments. It is interesting to notice that, our method produces competitive or often better PSNR results than other methods. As for the average PSNR result, the proposed method gains 0.19dB over the second-best IPAR algorithm. And for *Lighthouse*

and *Barbara*, the proposed method appears much better than NEDI, SAI and IPAR algorithm.

Table 2 PSNR(dB) results of five interpolation methods

Images	Bicubic	NEDI	SAI	IPAR	Proposed
Ruler	11.98	11.49	11.37	11.81	**12.43**
Slope	26.74	26.54	26.63	26.78	**27.14**
Rotate	29.75	29.25	30.79	33.15	**33.24**

Besides, on account of considering the multicollinearity, the proposed method achieves the desirable performance in interpolating piecewise smooth images. Table 2 shows the PSNR results of the five interpolation methods on three piecewise smooth images. The highest PSNRs demonstrate that our method generates much better results than other methods. Especially for *Ruler* and *Slope*, our method gains 0.45dB and 0.36 dB over the second-best algorithm IPAR.
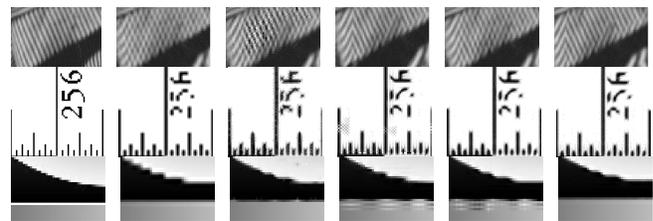
Fig.3 Visual comparisons: Portions from various interpolated images using different methods. From top to bottom: Barbara, Ruler and Slope. From left to right: ground truth, Bicubic, NEDI, SAI, IPAR, proposed method.

We also compare the visual quality of different interpolation methods. For *Slope*, it is obviously observed that Bicubic blurs the edge, and NEDI, SAI and IPAR produce annoying artifacts nearby the sharp edge. Due to considering the repetition of pixels along the isophote and the multicollinearity between model parameters, the results of our method reduce lots of artifacts and tend to be more similar to the original HR image. Moreover, for the images containing lots of edge structures and textures, our method produces fewer errors than others, like *ruler* and *Barbara* in Fig.3. It means that our method outperforms other methods in dealing with the high frequency part of images, such as edges and textures. More experimental results are released on our website[1].

5. CONCLUSION

In this paper, we propose a novel interpolation method based on the adaptive window-extension AR model combined with explicit interpolation methods. The model considers the local structural variation, estimates the isophotes with explicit interpolation method, and extends the window in the direction of the isophote. The piecewise stationarity of images signals is characterized by the patch-geodesic distance based similarity. And the precision of parameter estimation is raised by weighted ridge regression. Experimental results show that the proposed method achieves better performance.

---

6. REFERENCES

[1] Qing Wang and Rabab Kreidieh Ward, "A new orientation adaptive interpolation method," *IEEE Transactions on Image Processing*, vol. 16, no. 4, pp. 889–900, 2007.

[2] Christine M. Zwart and David H. Frakes, "Segment adaptive gradient angle interpolation," *IEEE Transactions on Image Processing*, vol. 22, no. 8, pp. 2960–9, 2013.

[3] Wenhan Yang, Jiaying Liu, Mading Li and Zongming Guo, "General Scale Interpolation Based on Fine-Grained Isophote Model With Consistency Constraint," *IEEE International Conference on Image Processing* (*ICIP*), Paris, Oct. 2014.

[4] Xin Li and Michael T. Orchard, "New edge-directed interpolation," *IEEE Transactions on Image Processing*, vol. 10, no. 10, pp. 1521–7, 2001.

[5] Lei Zhang and Xiaolin Wu, "An edge-guided image interpolation algorithm via directional filtering and data fusion," *IEEE Transactions on Image Processing*, vol. 15, no. 8, pp. 2226–38, 2006.

[6] Jie Ren, Jiaying Liu, Wei Bai, Zongming Guo, "Similarity modulated block estimation for image interpolation", *IEEE International Conference on Image Processing* (*ICIP*), Brussels, 2011.

[7] Mading Li, Jiaying Liu, Jie Ren, and Zongming Guo, "Adaptive general scale interpolation based on similar pixels weighting," *IEEE International Symposium on Circuits and Systems* (*ISCAS*) , Beijing, pp. 2143–2146, 2013.

[8] Tang Ketan, Oscar C. Au, Guo Yuanfang, Pang Jiahao, and Li Jiali, "Arbitrary factor image interpolation using geodesic distance weighted 2d autoregressive modeling," *IEEE international Conference on Acoustics, Speech and Signal Processing* (*ICASSP*), Vancouver, pp. 2217-2221, 2013

[9] Jian-Wu Xu, Shipeng Yu, Jinbo Bi, Lucian Vlad Lita, Radu Stefan Niculescu and R. Bharat Rao." Automatic Medical Coding of Patient Records via Weighted Ridge Regression." *International Conference on Machine Learning and Applications* (*ICMLA*), pp.260-265, Ohio, Dec. 2007.